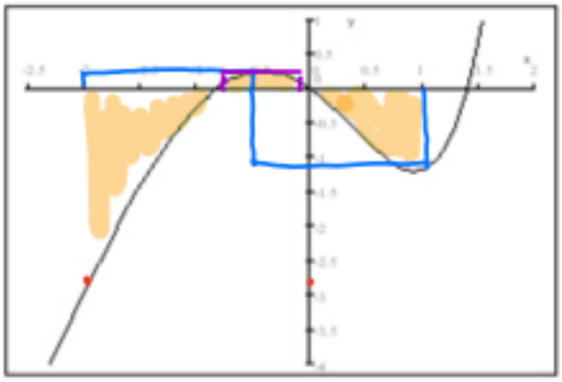
5.5 Consider the graph below.



$y = g(x)$

(a) Estimate the right Riemann sum approximation to $\int_{-2}^{1} g(x)\ dx$ using two subdivisions.

(b) Estimate the left Riemann sum approximation to $\int_{-2}^{1} g(x)\ dx$ using two subdivisions.

(c) Estimate the true value of $\int_{-2}^{1} g(x)\ dx$.

1.5   1.5

-2 (x_0) $-\frac{1}{2}$ (x_1) 1 (x_2)

$$\text{Right}(2) = \sum_{k=1}^{2} g(x_k)\Delta x = g(x_1)\Delta x + g(x_2)\Delta x$$

$$= \underset{(0.25)}{g(x_1)}(1.5) + \underset{(-1.1)}{g(x_2)}(1.5) = (-.85)(1.5)$$

$$\approx -1.3$$

$-.85$
$-.425$
-1.275

$$\text{Left}(2) = g(x_0)(1.5) + g(x_1)(1.5)$$

$$(-2.8)(1.5) + (0.25)(1.5) \simeq (-2.55)(1.5)$$

$$=$$

Practice:

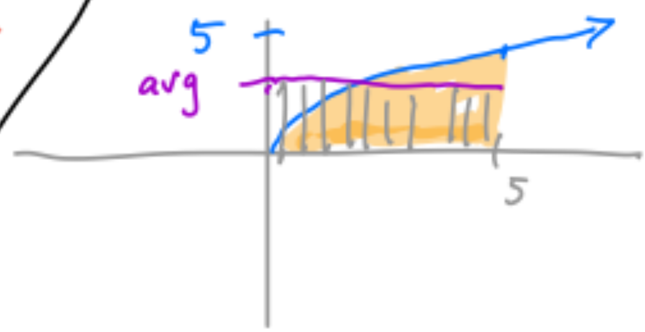
① Find the average value of $\sqrt{5x}$ on the interval $[0,5]$.

② Find $\displaystyle\int_1^4 \frac{dt}{t\sqrt{t}}$

③ Find $\displaystyle\int \frac{c}{1+c^4}\,dc$

---

① $$\underbrace{\frac{1}{b-a}\int_a^b f(x)\,dx}_{\text{average value of } f(x) \text{ on } [a,b].} = \frac{1}{5}\int_0^5 \sqrt{5x}\,dx$$



$(AB)^c = A^c B^c$

$\sqrt{AB} = \sqrt{A}\,\sqrt{B}$

$$= \frac{1}{5}\int_0^5 \sqrt{5}\,\sqrt{x}\,dx = \frac{\sqrt{5}}{5}\int_0^5 x^{1/2}\,dx$$

$$= \frac{\sqrt{5}}{5}\cdot\frac{2}{3}\cdot x^{3/2}\Big|_0^5$$

$5^{1/2}\cdot 5^{3/2} = 5^{1/2+3/2} = 5^2$

$$= \frac{\sqrt{5}}{5}\cdot\frac{2}{3}\cdot 5^{3/2} - 0 = \frac{5^{1/2}}{5^1}\cdot\frac{2}{3}\cdot 5^{3/2}$$

$$= \frac{5}{\cancel{8}} \cdot \frac{2}{3} = \frac{10}{3} = \boxed{\frac{10}{3}}$$

② $\displaystyle\int_1^4 \frac{1}{t\sqrt{t}}\,dt = \int_1^4 t^{-3/2}\,dt$

$t\sqrt{t} \to t^1 t^{1/2} = t^{3/2}$

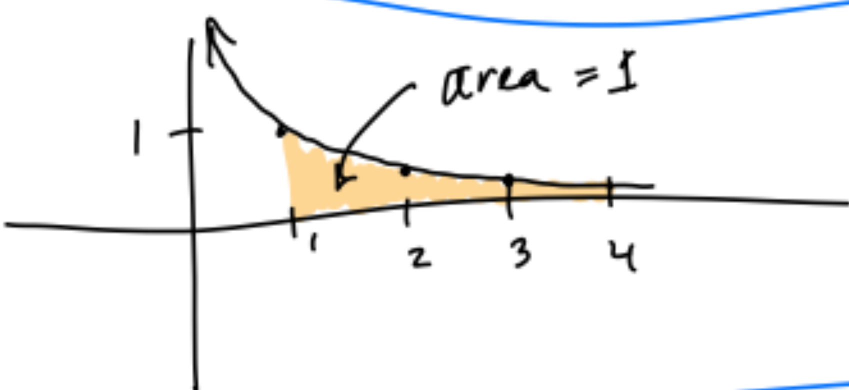
$$= -2t^{-1/2}\Big|_1^4$$

Check: $(-2t^{-1/2})' = -2\cdot\left(-\frac{1}{2}\right)t^{-3/2} = t^{-3/2}$ ✓

$$= -2\cdot 4^{-1/2} - \left(-2\cdot 1^{-1/2}\right)$$

$$= -2\cdot\frac{1}{4^{1/2}} + 2 = -1 + 2 = \boxed{1}.$$

$4^{1/2} = \sqrt{4} = 2 \qquad 1^{-1/2} = 1$



③ Find $\displaystyle\int \frac{c}{1+c^4}\,dc$

let $u = c^2$

$du = 2c\,dc$

$\frac{1}{2}du = c\,dc$

~~let $u = 1+c^4$~~

~~$du = 4c^3\,dc$~~ ← we don't have it.

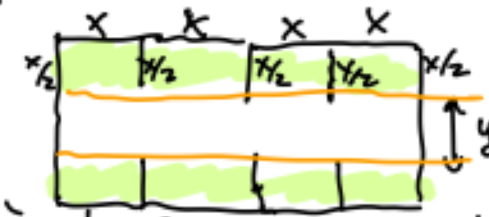
$$\int \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\arctan(u) + K = \boxed{\frac{1}{2}\arctan(c^2) + K}$$

constant

Check: $\left[\frac{1}{2}\arctan(c^2)\right]' = \frac{1}{2}\left(\arctan(c^2)\right)'$

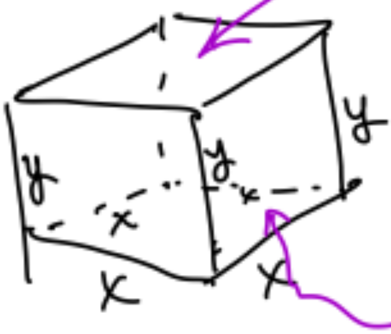
$$= \frac{1}{\cancel{2}}\,\frac{1}{1+(c^2)^2}\cdot\cancel{2}c = \frac{c}{1+c^4}. \checkmark$$

Ex To make a box out of cardboard, one takes a rectangle like this:



Cut along the black lines & fold to make.

twice as much cardboard



Question: If we want the box to have a volume of 2000 cm^3, how should we design the box so it uses the least amount of cardboard?

Volume $= 2000 = x^2 y$ $\Rightarrow y = \frac{2000}{x^2}$

Function = area of cardboard.

$$= \underbrace{4x^2}_{2\left(4x\cdot\left(\frac{x}{2}\right)\right)} + 4xy$$

$$F(x) = 4x^2 + 4x \cdot \frac{2000}{x^2}$$

$$F(x) = 4x^2 + \frac{8000}{x} = 4x^2 + 8000x^{-1}$$

minimize $F(x)$ on interval

$0 \leq x < \infty$

$$F'(x) = 8x + -8000x^{-2}$$

$$= 8x - \frac{8000}{x^2} = \frac{8x^3 - 8000}{x^2}$$

$$= 0$$

$$\Longleftrightarrow 8x^3 - 8000 = 0$$

$$\Longleftrightarrow x^3 - 1000 = 0$$

$$\Longleftrightarrow x^3 = 1000$$

$$\Longleftrightarrow x = 10$$

$F'(x) = - - - - \; 0 + + + + +$

x: 0 ... 10 ... ∞

$\therefore F(x) =$ must be abs. min

$\therefore \boxed{x = 10}$ What is y? $y = \frac{2000}{x^2} = \frac{2000}{100}$

best design: $\boxed{y = 20}$

In this list of integrals,

ⓐ Can it be done?

ⓑ If so, how would you do it?

	ⓐ		ⓐ
① $\int x e^{x^2} dx$	Y	⑤ $\int \sqrt{\cos(x)}\, dx$	N
② $\int x^2 e^{x^2} dx$	N	⑥ $\int \csc(p)\cot(p)\, dp$	Y
③ $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$	Y	⑦ $\int \csc(p)\sin(p)\, dp$	Y
④ $\int \cos(\sqrt{x})\, dx$	?(N)	⑧ $\int \cos^2(y)\sin(y)\, dy$	~~N~~ Y
		⑨ $\int \cos^2(y)\, dy$	~~N~~ Y

	ⓐ		ⓐ
① $\int x e^{x^2} dx$	Y	⑤ $\int \sqrt{\cos(x)}\, dx$ prob. not ??	N
ⓑ subs $u = x^2$			
② $\int x^2 e^{x^2} dx$	N ✓	⑥ $\int \csc(p)\cot(p)\, dp$ $= -\csc(p) + C$	Y
③ $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$ ⓑ subs $u = \sqrt{x}$	Y	⑦ $\int \csc(p)\sin(p)\, dp$ $= p + C$	Y
④ $\int \cos(\sqrt{x})\, dx$	?(N)	⑧ $\int \cos^2(y)\sin(y)\, dy$ let $u = \cos(y)$	~~N~~ Y
		⑨ $\int \cos^2(y)\, dy$	~~N~~ Y

let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

let $x = u^2$

$u \cos(u)$

integration by parts

$\frac{1}{2} + \frac{1}{2}\cos(2y)$

integrate

$\boxed{\frac{y}{2} + \frac{1}{4}\sin(2y) + C}$

Last Quiz of the Semester.

① What is your name?

② What will you the day after final exams are over?

③ Find the derivative.

ⓐ x^2 ⓒ $\cos(x)$ ⓔ $\arctan(x)$

ⓑ e^x ⓓ e^{2x} ⓕ $1+e^x$.